

 $\star \star$ 1. What are the next two numbers in this pattern?

0, 1, 1, 2, 3, 5, 8, ____, ____

 \star 2. Arrange the digits 1, 2, 3, 4, and 5 in the boxes so that the sum of the digits is the same in both directions.



Can you find another arrangement? What are possible sums?

 $\star \star \star$ 3. A toy shop makes tricycles and four-wheel wagons. Seven customers ordered six items each. Every order was different. How many wheels were needed for each customer?





 $\star \star \star$ 4. Use 17 toothpicks to make this figure:

Remove 6 toothpicks leaving 2 squares.

Strategy of the Month

Someone said, "A picture is worth a thousand words." Turning the words of a problem into a picture or a diagram can help you "see" the problem. By using the part of your brain that visualizes a situation or object, you may see relationships or information that helps you solve the problem. When someone tells you a story, try turning the words into a motion picture or a cartoon. When reading a description, try "seeing it in your mind's eye." If you can do these things, this strategy may be for you! Try using a picture or make a diagram to solve this problem:

In the restaurant there are 12 square tables. Only one person can sit on each side. What is the greatest number of people that can be seated if the tables are pushed end to end into one large rectangle?

Every year you grow and change in many different ways. Get someone to help you measure and record these data about yourself. Be sure to save the information because we will measure again in two months!

How tall are you?

How much do you weigh? _____

What is the circumference of your head?

 $\star\star$ 5. According to the graph below, what building is 20% taller than a 25 ft. house?



 $\star \star \star \star$ 6. Complete the pattern by filling in the missing numbers.

Χ	Y
2	1
4	5
6	9
8	
10	

Analyze the table and write a rule for the table so that you could find Y for any given X:

★★ 7. If a square is cut along one of its diagonals, two polygons of equal area are formed.
Will this also be true of a regular pentagon? Draw and explain your answer.

★★ 8. Add, subtract, multiply, and/or divide the numbers shown to get an answer of two. You may change the order, but you must use every number once and only once. Write an equation (number sentence) to show how you got your answer.

10 8 7 6 4

Setting Personal Goals

Problem solving is what you do when you don't know what to do. Being a good problem solver will help you be ready to live and work in our changing world. Computers can do computations but people must tell the computers what to do. Good problem solvers know how to make plans and use many different strategies in carrying out their plans. They use all of their past experiences to help them in new situations. We learn to swim by getting in the water; we learn to be good problem solvers by solving problems!



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Discussion of problems.....

1. (13, 21) This is known as the Fibonacci sequence. The pattern is derived by adding the first two numbers to obtain the third, e.g., 0+1 = 1; 1+1 = 2; 1+2=3; etc.

2. (one possible solution: $\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ Possible sums are 8, 9, and 10 2

3. (customer #1: 24, customer #2: 23, customer #3: 22, ... customer #7: 18, or vice-versa) A good strategy for this problem would be to make a table or an organized list, showing the combinations of items to make each order, totaling six items for each customer, and then computing the wheels needed for each.

	Tricycles	Wagons	Wheels
A similar list could be made	6	0	18
showing sides on squares	5	1	19
and triangles.	4	2	20
	3	3	21
	2	4	22
	1	5	23
	0	6	24



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- 5. (the hotel) 20% of 25 = 5; 5 + 25 = 30.
- 6. (13, 17; y = 2x 3)
- 7. (No) In a square, the diagonal is also a line of symmetry. Regular pentagons have no lines of symmetry that pass through two vertices. Therefore, diagonals will always form polygons of different sizes.



8. Possible answers: 10/(6+4) + (8-7) = 2 $10+6-(8/4 \times 7) = 2$ $(10-7) \times (8-6) - 4 = 2$

This question can easily become an ongoing game in the classroom using regular playing cards. Have the students draw a card to be a "target" answer. The next five cards drawn become the problem numbers to be used to reach the target. Let an Ace equal 1, a Jack equal 11, a Queen equal 12, and a King equal 13. All other cards count as face value. The game can be played as a whole class, group, partner, or individual activity.



★★ 1. Write a number in the \land that will make the answer 52.



 \star 2. WXYZ is a square. Other lengths are shown. Find the total area of the two shaded regions.



★ 3. In five days, how many times would a clock show 11:30?

 $\star \star \star$ 4. Hannah sold \$65 worth of barbecue tickets. Adult tickets cost \$4 each and children's tickets cost \$3 each. How many adult tickets could Hannah have sold? Is there more than one possible solution to this problem?

Strategy of the Month

Your brain is an organizer. It organizes information as it stores that information. When a problem involves many pieces of information, your brain will have an easier time sorting through it if you make an organized list. A list helps you be sure you have thought of all of the possibilities without repeating any of them. Like drawing a picture or making a diagram, making an organized list helps your brain "see" the problem clearly and find a solution. Try **making an organized list** to solve this problem:

Tickets for the concert cost \$12 for adults or teenagers and \$6 for children. If the group has \$60, how many adults or teenagers and how many children could go? MathStars Home Hints Sometimes the hardest part of solving a problem is just getting started. Having some steps to follow may help you. 1. Understand the information in the problem and what you are trying to find out. 2. Try a strategy you think might help you solve the problem. 3. Find the solution using that strategy or try another way until you solve the problem. 4. Check back to make certain your answer makes sense.

 $\star \star \star$ 5. Draw the next figure in the pattern. How many circles are needed?



How many circles would be needed for the ninth figure?

★ 6. How many toothpicks (*edge*) and gumdrops (*vertices*) are needed to build a square pyramid? Draw a sketch of your answer.

 $\star \star \star$ 7. Draw two mirror images of the design: one on the right and one below it. Use these new drawings to draw the mirror image in the lower right space. Draw dotted lines to show all lines of symmetry in the completed drawing.







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Discussion of problems.....

1. (23) Have students work backward and reverse the operations.

 $52 + 7 = 59 \ge 2 = 118 - 3 = 115 \div 5 = 23$

- 2. (84 sq. units) The total area of the square is $12 \times 12 = 144$ square units. The unshaded areas equal $(12 \times 2) + (12 \times 1) + (12 \times 2) = 24 + 12 + 24 = 60$ sq. units. The remaining shaded area is 144 60 or 84 sq. units
- 3. (10 times) Each day the clock shows each time twice, as it makes two complete 12 hour cycles.
- 4. **(possible answer: 14; there are several correct solutions)** To solve this type of problem, students could organize information in a table, such as:

Adult tickets	Child tickets	Total cost
14 @ \$4 (\$56)	3 @ \$3 (\$9)	\$65
11 @ \$4 (\$44)	7 @ \$3 (\$21)	\$65
8 @ \$4 (\$32)	11 @ \$3 (\$33)	\$65
5 @ \$4 (\$20)	15 @ \$3 (\$45)	\$65
2 @ \$4 (\$8)	19 @ \$3 (\$57)	\$65

The strategy here is to notice that the odd multiples of 3 form a pattern of odd numbers and the multiples of 4 form a pattern of even numbers, and that certain ones combine to give a sum of \$65. Notice that not all odd multiples of 3 will work, e.g., $5 \times 3 = 15$; 15 from 65 gives 50, which is not a multiple of 4, so 5 child tickets would not be possible. Likewise, all even multiples of 3 will result in an even product, and when subtracted from 65 will result in an odd number, which is not a multiple of 4. Notice also that the numbers of adult tickets decrease by 3 in that column, while the corresponding numbers of child tickets increase by 4.

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5. (16 circles are needed for the next figure. 61 circles are needed for the ninth figure.)



- 6. **(8 toothpicks and 5 gumdrops)** Modeling with toothpicks and gumdrops (or marshmallows or jellybeans) emphasizes the differences between vertices and edges. Allow students time to experiment with making their own 3-D figures as well as assigning them specific items to make.
- 7. Have students use thin tracing paper to practice with lines of symmetry and mirror images. Folding this paper allows the students to see through the paper to check their work.





★ 1. What number would come next in the sequence?

81 27 9 3 1 ____

★★ 2. A jigsaw puzzle has 289 pieces. Each piece is basically square. How can it be arranged so that every row and column of the completed puzzle has exactly the same number of pieces?

★★ 4. Marcus got 37 hits in 46 times at bat during one Little League season. Laverne got 23 hits in 31 times at bat. Who was the better hitter? Explain.



 $\star \star \star \star$ 3. Jake earned \$576 during the month of February. He was paid \$6 per hour. He did not work more than five hours each day, nor did he work on Sunday. If he worked the same number of hours each day, how many hours per day did he work? How did you figure this out?

Strategy of the Month

Being a problem solver is something like being a detective! A detective has to solve crimes by guessing what happened and checking the guess to see if it fits the situation. For some problems, your best strategy may be to make a guess and then check to see if your answer fits the problem. If not, decide if your guess was too high or too low and then make a second "guesstimate." A good detective keeps records (usually some kind of chart) to help see any patterns and to narrow down the possibilities. You should do this too. The results of incorrect guesses can give you valuable clues to the correct solution. **Guess and then check** the solution to this problem:

Use exactly 50 coins to make one dollar. You must have at least one penny, one nickel, one dime, and one quarter.

Memorizing number facts will save you time.
Flash cards are one way to learn new facts, but you also might try these ideas:
play dice or card games in which you need to add, subtract, multiply, or divide.
learn new facts using ones you already know (7+7 =14 so 7+8=15).
learn facts that are related to each other (7x6=42, 6x7=42, 42÷6=7, 42÷7=6).
make a list of the facts you need to memorize and learn 5 new facts each week.
Spend 5-10 minutes every day practicing facts.

★★ 5. The Johnsons have three pigs. They keep them in three pens like the ones shown below. When they adopted a new pig, they did not have enough money to build another pen. How can they rearrange the three pens they have to make a fourth pen, keeping all pens the same size and shape?







★★★ 6. A train leaves Miami for Atlanta, 600 miles away, traveling at a rate of 125 miles per hour. At the same time another train leaves Atlanta for Miami, traveling at a rate of 75 miles per hour. When the two trains meet, which train is nearer to Miami?

★★ 7. From the bottom of a thirty-foot hole, a frog can climb up four feet each day, but slips back two feet each night. In how many days does the frog escape from the hole?

★ 8. On June 1, the temperature in Charlotte, North Carolina, was 83 degrees Fahrenheit. In Oslo, Norway, the temperature was -27 degrees Fahrenheit. What was the temperature difference?

Setting Personal Goals

Communicating mathematically means that you are able to share your ideas and understandings with others orally and in writing. Because there is a strong link between language and the way we understand ideas, you should take part in discussions, ask questions when you do not understand, and think about how you would explain to someone else the steps you use in solving problems.



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Discussion of problems.....

- 1. (1/3 or 0.33...) Divide by 3 each time.
- 2. (A 17 x 17 square) Trial and error work with factoring will lead students to determine that 289 is a square number.
- 3. **(4 hours)** \$576 divided by \$6 indicates Jake worked 96 hours. Factoring gives the following possible solutions: 2 x 48, 3 x 32, 4 x 24, 6 x 16, 8 x 12. Only four hours for 24 days meets all of the conditions.
- 4. **(Marcus)** Marcus' average was 0.804 while Laverne's was only 0.742. Have students find their own batting averages or kicking averages if they play kickball.
- 5. (See the drawing.) If needed, students can use triangle pieces and move these around to various positions. By sliding two pens together, corner-to-corner, and putting the third pen on top of the two, a fourth pen is created without building an extra pen.



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6. (**neither train**) In this non-routine problem, students must visualize the scenario of the two trains approaching, but realize that when they meet, both are the same distance from Miami. In this case, none of the numbers in the situations are needed to solve the problem. Students may wish to act it out, or use models to represent the two trains approaching each other.

7. (the fourteenth day) Have students draw a picture or diagram of the frog in the hole, and represent his daily progress. For 13 days, he makes a net gain of two feet per day, or 26 feet total. On the fourteenth day, he climbs up four feet, and is free!

8. (110 degrees) The student must first use the rules for determining the difference between positive and negative numbers, i.e., change sign of subtrahend and proceed as though it were addition. 83 - (-27) = 83 + 27 = 110



★★ 1. On Thursday the temperature was -10 degrees. On Friday it rose nine degrees. On Saturday it dropped 15 degrees and on Sunday it rose seven degrees. What was the temperature on Sunday?



★★ 3. A recipe for three quarts of punch calls for 1/2 cup of lemon juice. Your mom has only 1/3 cup of lemon juice. Does she have enough for two quarts of punch, one quart of punch, or would you have to give up the idea of making punch until she gets more lemon juice? Explain.

★ 4. John can cut a log into three pieces in 24 minutes. At this rate, how long will it take him to cut another similar log into eight pieces?

 $\star \star \star \star$ 5. At summer camp there were 200 Boy Scouts, divided into five groups. When they graphed the number of merit badges they had earned, one group made this graph:

Number of Merit Badges

Fewer than 5 badges: ፚ፞ፚፚፚፚፚፚ

5-12 badges: ជជជជជជជជជជជជជជជជជ

13-20 badges: ជំជំជំជំជំជំជំជំជំជំជំជំជំជំ

21 or more badges: ☆☆☆☆☆

How would you predict the total number of merit badges for all of the Scouts? What would your prediction be?

Strategy of the Month

Noticing patterns helps people solve problems at home, at work, and especially in math class! Math has been called "the study of patterns," so it makes sense to look for a pattern when you are trying to solve a problem. Recognizing patterns helps you to see how things are organized and to make predictions. If you think you see a pattern, try several examples to see if using the pattern will fit the problem situation. Looking for patterns is helpful to use along with other strategies such as make a list or guess and check. How can finding a pattern help you solve this problem?

A palindromic number is one which reads the same backwards as forwards. How many 3-digit palindromic numbers are there?

Set aside a special time each day to study. This should be a time to do homework, to review, or to do extra reading. Be organized and have a special place in which to work. This place needs to have a good light and to be a place where you can concentrate. Some people like to study with quiet music; others like to sit at the kitchen table. You need to find what works for you!

Remember that when you are reviewing or working on solving problems it may help to study in a group.

★ 6. If each letter in the word SUPER-STARS was written on a card and placed in a hat, what would be the probability of drawing

a. an R _____

- b. a vowel _____
- c. an R or an S _____

 $\star\star$ 7. How many right triangles can be formed by drawing all three lines of symmetry in an equilateral triangle?

Draw your answer. (You may use more than one drawing to make your answer clear.)

★★★ 8. Jan sat down to eat a whole a pizza. Barry asked for some, so Jan gave Barry half. Marcus also wanted pizza, so Jan gave Marcus half of what was left. Then Nina asked for pizza too, so Jan gave Nina half of what was left. Next, Demetrius asked for pizza, so Jan gave him half of the remaining pizza. How much pizza did each person get? Color and label their names on a circle graph to show your answer.



Setting Personal Goals

If your goal is to become a more <u>responsible</u> student, it means that you:

- actively participate in class.
- complete your assignments.
- have everything you need in class.
- ask for help when you do not understand.
- be willing to investigate new ideas.



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Discussion of problems.....

- 1. (-9 degrees) Using integers to represent the changes in temperature gives +9, -15, and +7. The sum of these is 1. -10 + 1 = -9
- 2. **(63, 127)** The pattern increases by double the amount of the previous difference, e.g., +2, +4, +8, +16, +32, etc.
- 3. (two quarts of punch) Strategy logic/reasoning:
 - a. convert the fractions 1/2 and 1/3 to common denominator, 3/6 and 2/6.
 - b. equate the numerators to the number of quarts, i.e., 3 = 3 qts., 2 = 2 qts. The numerator 2 equals the number of quarts of punch that can be made.
- 4. (84 minutes) Strategy: To find the number of minutes per cut, divide the 24 minutes by 2, as it takes 2 cuts to cut a log into 3 pieces (12 minutes each). Then multiply by 7, since 7 cuts are needed to cut a log into 8 pieces ($12 \times 7 = 84$). Students may also wish to draw a picture of the log, showing the 2 cuts needed for 3 pieces, then another log, with 7 cuts for 8 pieces, labeling each cut with 12 minutes.





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5. (2000 – 3000 is a reasonable range) Strategy: Using an approximate average of about 12 badges per scout in the total group shown on the graph, a reasonable estimate for the total group would be in the area of 2400 total, or in the range of 2000 - 3000.

To find the average of the group in the graph, one technique would be to multiply the number of scouts in each category by a "middle" number in that category, add the total for each category, and divide by the number of scouts represented. For the first category, "fewer than 5 badges", one might multiply the number of scouts shown (8) by 3, for a total of 24 badges in that category. In the next category, "5 to 12 badges", you could multiply 14 scouts by a "middle" number 9, for a total of 126 badges in that category; then proceed to 12 scouts times 17 badges for 204, and 5 scouts times an estimated 22 badges for 110. The total badges for this group of 39 scouts is about 464, or an average of nearly 12 per scout. If this group, approximately one-fifth the total group, is typical of the whole group, the total number of badges might be projected to be about 2320, or 5 times the number in the first group.

- 6. (For A: 2/10 or 1/5. For B: 3/10. For C: 5/10 or 1/2) Encourage students who may have difficulty to actually write the letters on cards or in separate columns on their paper to clarify their choices.
- 7. (12: See the drawing) Encourage students to sketch their drawings on separate triangles. Remind them to use the corner of a sheet of paper as a quick check for right angles, so they won't get bogged down in measuring with protractors.



8. **(See the circle graph.)** Allow students to experiment with fraction circles to explore the effects of the clues.





 $\star \star \star$ 1. If a square pyramid is placed on top of a cube, how many faces, vertices, and edges will the new geometric solid have? (Assume the square bottom of the pyramid is the same size as a face on the cube.) Illustrate.

 $\star \star \star \star$ 4. A jigsaw puzzle has 50 border pieces and other non-border pieces. If each piece is one unit in length, how many units wide and how many units long could the puzzle be?

Is there more than one possible answer? Explain.

★★ 2. If size 6 pants require 25 inches of elastic and size 10 pants require 30 inches of elastic, how much elastic would size 12 pants need?

 \star 3. What number is missing in the sequence?

12 7 2 _____ -8

Strategy of the Month

Sometimes mathematical ideas are hard to think about without something to look at or to move around. Drawing a picture or using objects or models helps your brain "see" the details, organize the information, and carry out the action in the problem. Beans, pennies, toothpicks, pebbles, or cubes are good manipulatives to help you model a problem. You can use objects as you guess and check or look for patterns. Try **using objects** to help you solve this problem:

What happens to the volume of a rectangular prism if the width is tripled?

Remember when you had "Show and Tell" in kindergarten? Now you have a great deal to share in mathematics. Talk to the folks at home about what you are learning. Show them your papers and tell them about what is happening in your math class. Let them see that you are doing problems in class similar to these. Each week choose an assignment that you are proud of and display it somewhere in your house.

 $\star \star \star$ 5. A zookeeper is ordering food for the zebras. She knows that three zebras eat 25 pounds of hay every three days. How much hay should she order for 12 zebras to have enough hay for 30 days?

★★ 8. At the ball game, Lucy bought two hamburgers, two hot dogs, four fries, three soft drinks, and a milkshake. How much change did she receive from her \$20 bill?

Menu			
Hamburger	.85	Milk .50	
Hotdog Grilled Chees	.70 se .95	Soft Drink Milkshake	.75 .95
French Fries	.60	Ice Cream	.55

 \star \star 9. Draw this figure to show it reflected along the dotted line.



 \star 6. Without using pennies, how many ways can you make change for a half dollar? Explain.

 $\star \star \star$ 7. Find the quotient and remainder of 179 divided by 11 on a calculator without using the division key. What is the quotient and remainder? Explain.

Setting Personal Goals

Mathematics is all around us. We use it every day in personal living and in all of our school work. When we read graphs in social studies, gather and use data in science investigations, or count in music or physical education, we are using mathematics. We make connections in our math classes also; for example, measurement skills help us in solving many geometry problems and classification skills help us in organizing data. We use computation in many different situations. You will become a stonger mathematics student by making connections.



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Discussion of problems.....

- 1. (Faces = 9, vertices = 9, edges = 16) Allow students to experiment with geometric solids, combining different shapes to create new figures.
- 2. (32 1/2 inches) From size 6 to size 10 is a difference of 2 sizes and 5 inches. The difference for one size, therefore, is 2 1/2 inches. Students may consider the change from size 6 to size 10 as four sizes, which will equal 1 1/4 inches per size. Either interpretation produces an increase of 2 1/2 inches to change from size 10 to size 12.
- 3. (-5) Subtract 5 each time. Using the constant key on a calculator can help students experiment with patterns in addition, subtraction, multiplication, and division.
- 4. (Possible answers: 14 x 13, 15 x 12, 16 x 11, 17 x 10, 18 x 9, 19 x 8, 20 x 7, 21 x 6, 22 x 5, 23 x 4, 24 x 3) The corner pieces of the puzzle provide length and width in the same piece. Therefore, the perimeter actually equals 54 units, but is constructed with 50 pieces.
- 5. (1000 pounds of hay) Students can reason proportionally that if 3 zebras eat 25 pounds of hay in three days, then 12 zebras will need 100 pounds in three days. Thirty days is ten groups of three and so 10 x 100 or 1000 pounds of hay are needed.
- 6. (10 ways) Encourage students to make a chart, showing a column for each type of coin:

Quarters	Dimes	Nickels
2	0	0
1	2	1
1	1	3
1	0	5
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10



9.

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- 7. (Answers may vary) Students may subtract 11 from 179 repeatedly, until what remains is less than 11. The quotient is the number of times 11 was subtracted, and the remainder is what is left on the calculator display.
 Another method would be to add 11 repeatedly, keeping track of the number of times it is added without going over 179. The quotient is the number of 11's added, and the remainder is the difference between the sum and 179.
- 8. (\$11.30) Have students use the price list. When multiple items are purchased, multiply, i.e., 4 fries x .60 = \$2.40. Continue with this procedure for each item. Add the total of each group of items to get the total amount spent (\$8.70). Subtract this from \$20.00.





 $\star \star \star$ 1. Jason has two footballs and a helmet that cost him \$40. Scott has two helmets and a football that cost him \$47. What is the cost of each item?



★★ 2. Andrea's 4-H club is planning a rock-athon. The members have agreed to rock in rocking chairs for one full week in order to raise money for their calf project. So far, club members have been rocking for 4 days, 17 hours, 36 minutes, and 9 seconds. How much longer do they need to continue rocking?

★ 3. Which of the following terms describes these two figures? Circle the letter beside the correct answer:



- a. congruent
- b. similar
- c. same symmetry and congruent
- d. similar and same symmetry

 $\star \star \star$ 4. Mr. Cooper's Carpenter Shop makes 3-legged stools and 4-legged chairs, using the same kind of legs. If Mr. Cooper has 98 legs on hand,

- a. how many of each kind can he make if he makes 28 seats altogether?
- b. how many different combinations of chairs and stools are possible?

He can make

a. ____ chairs and ____ stools using a total of 28 seats.

b. _____ different combinations are possible.

Strategy of the Month

When a problem involves data with more than one characteristic, **making a table, chart, or graph** is a very good way to organize the information. It helps your brain to identify patterns and to discover any missing data. Tables help you record data without repeating yourself. Making a table or chart is especially useful for certain problems about probability and for some logic problems. Sometimes tables and charts are included in your information and you need to read through them carefully to understand the data you need to solve your problem. Creating a graph is also a good way to organize and visualize information. **Make a table** to solve this problem:

Farmer Oakes had 15 animals in her farmyard. Some were chickens and some were cows. There were 52 legs in all. How many cows were in her farmyard?

Everyone learns from sharing, and you can continue to learn by teaching others about the new mathematics ideas you are learning. Become a teacher and help a younger student. Explain what you have learned and what else you want to know. Good teachers set goals and evaluate the progress made toward reaching these goals. You will continue to be a learner whenever you become a teacher.

★ 5. Cut a square out of paper. Fold each corner to the center to form a second square. (See the illustration.) How does the area of the second square compare to the area of the first square?



 $\star \star \star$ 6. What happens to the area of a triangle if the height is doubled?

 \star 7. Boris spends one and a half hours a day studying. Shade the circle graph to show how his study time compares with other activities.



★ 8. Try this without pencil and paper or calculator!

81 + 41 + 86 - 39 - 2 = _____

★★ 9. The dieter opened a new box of chocolate cookies. He immediately ate half of them. That night he ate half of what was left. For a mid-morning snack the next day, he ate one-fourth of what was left. At lunch, he ate one-third of what was left. At 3:00 p.m. he ate half of what was left and polished off the last cookie right before dinner. How many cookies were in each box?

Setting Personal Goals

Perseverance means that you do not give up easily. Good problem solvers try different strategies when they are stumped and are not discouraged when they cannot find an answer quickly. They stick to the task, using all of their previous experiences to make connections with what they know and the problem they are trying to solve. If something does not work, they discard the unsuccessful idea and try again using a different strategy.



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Discussion of problems.....

1. (footballs: \$11; helmets: \$18) Students may guess and test, or make a table something like

this:		Footballs	Helmets	Cost
	Jason	2@\$11	1@\$18	\$40
	Scott	1@\$11	2@\$18	\$47

2. (2 days, 6 hours, 23 minutes, 51 seconds) To solve this students must convert one week into 7 days, then convert one day into hours, minutes, and seconds, then proceed to subtract, regrouping as necessary:

	6 days,	23 hours	, 59 minutes, and	60 seconds
-	4	17	36	9
	2 days	6 hours	23 minutes	51 seconds

- 3. (**d. similar and symmetrical**) Students may need to review the meaning of the terms. The figures are not congruent because they are not the same size. However, they both are the same shape and both are symmetrical.
- 4. (a. 14 chairs, 14 stools; b. 8 combinations) Since 98 is not divisible by either 3 or 4, Mr. Cooper could not make all chairs or all stools, but must have a combination of them to use all of the legs. The maximum number of chairs is 23, with 2 stools; the maximum number of stools is 30, with 2 chairs. Students may guess and test to find combinations that will fit the criteria, but a chart or table would be useful in finding all of them.

# of chairs	x 4 legs	# of stools	x 3 legs	Total legs used
23	92	2	6	98
20	80	6	18	98
17	68	10	30	98
14	56	14	42	98
11	44	18	54	98
8	32	22	66	98
5	20	26	78	98
2	8	30	90	98



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- 5. (The second square is 1/2 of the area of the first.) Many origami activities can be used to illustrate and reinforce geometric concepts. The given fold is the first step in making a samurai warrior. It is also the same step in making a paper "fortune teller" which many students play with.
- 6. (The area is doubled.) The area of a triangle equals $1/2 \ge b \ge h$. If the height is doubled, the area becomes:

 $\frac{1}{2} \ge b \ge 2 \ge h =$ 1 \empty b \empty h or just b \empty h

Work with geoboards and dot paper can help students develop understanding of this formula.

7. (Approximate. See the circle graph.) 1 1/2 hours is 1/16 of 24 hours.



- 8. (167) Students should recognize that adding 41 is the opposite of (-39 -2)
- 9. (16 cookies) Students will probably work backwards to determine the solution.

<u>TIME</u>	# OF COOKIES DIETER HAD
Before dinner	1
3:00 p.m.	2
Lunch	3
Mid-morning	4
Night before	8
New box	16



★ 1. If four people each eat a slice of the mushroom pizza and five people each eat a slice of the pepperoni pizza, which pizza will have the smaller amount left over?



 $\star\star$ 4. Draw the next two items in the pattern sequence. Label the perimeter for each.



★★ 2. Latrice earns tips for waiting on tables at a restaurant. During one 8-hour shift, she noticed that her tips increased by \$1 during each additional hour of work. If she earned \$2 the first hour, when did her income equal or exceed the hostess's pay of \$4.25 per hour?

★ 3. A store advertises shirts as "Buy one and get the second one 50% off." If the shirts were originally \$19.00 each, what is the average discount for each shirt?

Strategy of the Month

Some problems are difficult to "see" even if you draw a picture. For these problems, it can be helpful to actually **act out the problem**. When you role play with friends or people at home, you may discover the solution as you act out the problem. Or you may recognize another strategy that will help you find the answer. Sometimes "acting out" a problem can be done with manipulative materials. To find the solution to the problem below, become the director and choose your cast to act this out:

The students were in line at the movie theater to buy tickets. There was a student in front of two students, student between two students, and a student behind three students. What is the least number of students that could have been in line?

Calculators are important tools. They do not replace mathematical thinking; you must tell the calculator what numbers and operations to use. Calculators allow students to focus their energies on solving problems and to easily try alternative solutions. They also allow students to solve problems that were too difficult for pencil and paper. Number sense and good estimation skills are important when students use technology to carry out computations. Explore some "what if" situations with the calculator. "What if the cost of gas goes up $4 \notin$... What if we build the patio 2 feet wider..."

 $\star \star \star$ 5. Some unicycles, bicycles, and tricycles are parked in a parking lot. There are 65 wheels in all. How many of each type of vehicle are in the lot?

★★ 7. Team managers predict a crowd of about 2500 for Friday's football game. About how many packages of cups should the concession stand manager plan to order, if the cups come five dozen to a package? Explain how you estimated the number of packages.

★★ 8. In the product $4 \times 5 \times 6 \times 7 \times 8 \times 9$, which of the six numbers should be increased by one to cause the greatest increase in the product?

 $\star \star \star \star$ 6. Tommy's pet mouse, Spooky, is 20 centimeters long. Spooky's head is 1/4 as long as the main section of his body. The tail of the mouse is as long as its head and body section combined. How long is each part of Spooky's body:

★ 9. The product of 5 and a number is 5/8. What is the number?







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Discussion of problems.....

- 1. (**Mushroom pizza**) 1/3 of the mushroom pizza will be left over after 4 slices are eaten. 1/2 of the pepperoni pizza will be left over after five slices are eaten. Suggestion: Have the students color the parts eaten to distinguish the relative sizes of the leftovers.
- 2. (**During the 6th hour**) Latrice's earnings at the end of each hour total 2, 5, 9, 14, 20, 27. The hostess will earn more until then, when her income will be \$25.50.
- 3. (25% off or \$4.75 off) Students can approach the problem by finding 50% off one shirt and dividing by two or by dividing to 50% off by two to get 25% off each.
- 4. (**Perimeters: 14, 16. See the drawings.**) Provide access to pattern blocks if possible. Students should be able to determine that the side of the triangle equals one unit of the perimeter.

$$\begin{array}{c} \hline \\ P = 14 \end{array} \qquad \qquad \begin{array}{c} \hline \\ P = 16 \end{array}$$



5. (Answers may vary)

65 unicycles or 21 tricycles and one bicycle would have the correct number of wheels; however, the problem states that unicycles, bicycles, and tricycles are parked in the lot. The greatest number of tricycles possible would be 20 with one unicycle and two bicycles or 20 tricycles, one bicycle, and three unicycles. Instead of using three wheels for a tricycle, one bicycle and one unicycle or three unicycles would have a total of three wheels. Students may list possible answers in a chart such as the following:

BICYCLES	UNICYCLES
2	1
1	3
3	2
2	4
1	6
	BICYCLES 2 1 3 2 1

6. (H = 2, B = 8, T = 10) This problem can be solved by using the head as a unit of measurement. H + HHHH + HHHHH = 20 cm. 10H = 20; H = 2 cm.

7. (42 packages if student estimates one drink per person; answers may vary)

To solve this estimation, first discuss how many people attending the game are likely to buy a drink. If students agree that an average of one drink per person is reasonable, that will equal about 2500 drinks. The next step is to convert dozens to number of cups per package: $5 \times 12 = 60$ cups; then divide 2500 by 60 to get the number of packages. The quotient is 41.667 pkgs. which must be rounded to 42 as you cannot buy a part of a package.

- 8. (4) 4 x (5 x 6 x 7 x 8 x 9) = (4 x 5 x 6 x 7 x 8) x 9. However, (5 x 6 x 7 x 8 x 9) is greater than (4 x 5 x 6 x 7 x 8). Increasing the 4 by one results in one more (5 x 6 x 7 x 8 x 9) rather than one more (4 x 5 x 6 x 7 x 8) which would result from increasing the 9 by one. Also: 5 x (5 x 6 x 7 x 8 x 9) = 75,600 (4 x 5 x 6 x 7 x 8) x 10 = 67,200
- 9. (1/8) Students should readily see that if 5 x = 5/8

= 1/8



 $\star \star \star$ 1. Find the least six-digit number that is divisible by 2, 3, 4, 5, 6, and 9. The number must not contain any digit more than once.

 $\star\star$ 4. A rectangular dog lot 10 feet wide has a diagonal of 26 feet. How much fencing would be needed to enclose it? Explain how you got your answer.





★ 3. What is the maximum number of triangles that you can make on this quadrilateral by drawing only two vertical lines?





Strategy of the Month

What do you do if you have a problem that seems to be very complicated? It may have a lot of large numbers, too much information, or multiple conditions. One approach is to create a simpler problem like the one you need to solve. As you solve the easier problem, you may see the way to solve the more difficult one. Or you may discover a different process that will work with the harder problem. The trick is to be sure that your simpler problem is enough like the original one that the patterns or process you use will help you with the harder situation. Make a simpler problem first as you solve this:

The houses on Cox Avenue are numbered consecutively from 101 to 950. How many house numbers contain at least one digit 5?

Math skills develop as you apply concepts learned in school to real life situations. Which product is the best buy? How many tiles will it take to cover the kitchen floor? What time should we start baking the turkey so that we can have dinner at 7p.m.? What do the statistics tell us about the two baseball players?

★★ 5. How many different ways can a square be formed by connecting four vertices at a time? Draw and label your solutions on the dots. (Use more than one set to make your answers as clear as possible.)



★★ 6. At the end of the game the players from two little league teams of 25 players each shook hands. Each player shook hands with all of the players on the opposing team once. How many handshakes were there? Explain how you know.

★★ 7. A store advertises shirts as "Buy one and get the second one 50% off." If the shirts were originally \$19 each, what is the average discount for each shirt?



★ 8. Find the numbers whose sum is 17 and whose difference is 5.

What is their product?

Setting Personal Goals Confidence means that you believe in yourself. You can become a more confident problem solver by learning to use a variety of strategies. If your first idea does not work, don't give up, just try another way! Working with a buddy also helps. You need to remember that there is usually more than one way to solve a problem and that practice always helps us learn.



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Discussion of problems.....

1. (123,480)

Because the number is even and divisible by five, the digit in the ones place must be 0. In order to be divisible by four the last two digits must by divisible by 4; therefore, the digits in the tens place can possibly be 2, 4, 6, or 8. In order to be divisible by 3, 6, and 9 the sum of the digits must be divisible by 9. Using this information and a "guess and check" procedure, students should be able to obtain the solution.

2. (25 floors)

Students may use positive and negative numbers to determine the solution. Each time the elevator goes up a positive number can be used and negative numbers can be used when the elevator goes down. Because the elevator started on the third floor the following equation would be appropriate:

3 + 8 + (-9) + 13 + (-4) + 14 = 25







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4. (68 feet) Strategy: draw a picture. When students can see the dimensions of the rectangular lot, then it is not difficult to figure the perimeter by adding the lengths of the four sides. To obtain the length of the sides of the rectangle, they must apply the Pythagorean theorem, which states that the sum of the square of the width plus the square of the length is equal to the square of the diagonal. (In a right triangle, $a^2 + b^2 = c^2$)



5. (20) See the drawings below. Work with geoboards will help with this type of problem. Note: Because grids are not drawn to scale figures do not appear to be squares. #1-10









- 6. (625) Students should have a clear explanation to earn credit for this problem. The 25 players on each team shake hands with players on the opposing team only, not with their own team members. Solving simpler problems is a goal strategy for this problem.
- 7. (25% off or \$4.75 off) Students can approach the problem by finding 50% off of one shirt and dividing by two or by dividing to 50% off by two to get 25% off of each.
- (66) 11 + 6 = 17; 11 6 = 5; $11 \ge 6 = 66$. 8.



★ 1. Will the number of faces, vertices, and edges change if the two items are joined end to end? How many faces, vertices, and edges does each solid have? How many faces, vertices, and edges will the new solid have if the two are joined end to end?



 $\star \star \star$ 2. How many fewer square centimeters are used by the smallest eyes on a standard stove as compared to the largest? (Do not measure the stove if the eyes are hot!)

 $\star \star \star$ 3. If a healthy cow produces about ninety 8-oz. glasses of milk a day, how many gallons would the cow produce in May? How do you know?

 \star 4. What is the narrowest angle that can be created by connecting three different dots?

Draw and label the measurement of your answer.



Strategy of the Month

What if you know the result of a situation, but you don't know the beginning? For example, you might know that you end up with thirteen baseball cards after doing a certain number of trades and you want to figure out how many cards you had before the trading started. In that case you need to work backwards; you have to think about your actions in reverse order. This strategy works for any sequence of actions when you know the end result rather than the starting place. Try **working backwards** to find the starting number on this flow chart:



Mathematics can make life easier for you when you become a good estimator. Spatial estimation helps you plan how you will rearrange your furniture or how far to jump to cross a puddle of water. Using estimation helps you know if you have enough money for your purchases before you get to the check-out line. We become good estimators by practicing. Use your number sense and spatial sense to think about what the answers to problems will be before you start to solve them.

 $\star\star$ 5. Triangles ABC and DEF are similar. Give the length of the missing side of triangle DEF.



 $\star \star \star \star$ 7. The Veteran's Day parade is exactly one mile long. If the parade route is exactly two miles long, and the parade is marching at a rate of four miles per hour, how long will it take the parade to completely finish the route?

★★★ 8. Benjie invited Travis to play a game with two dice. They were to roll the dice and multiply the two numbers shown. If the product was even, Benjie would get a point, and if the product was odd, Travis would get a point. Make a table or tree diagram to determine who was more likely to win the game, and explain why.



★★ 6. A clockmaker must wind his clocks on a regular schedule. He winds part of his clocks every two days, part of his clocks every three days, and part of his clocks every five days. How often must he wind all of his clocks on the same day?



Setting Personal Goals

When you encounter a new situation, you use all of your previous experiences to figure out the current problem. Reasoning mathematically means using your brain power to think logically and sequentially, to put prior knowledge with new information. Set the goal of developing mathematical power and use your thinking power to achieve the goal!



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Discussion of problems.....

- 1. (No; 6 faces, 8 verticies, 12 edges) Provide building blocks for experimentation with these and other 3-D solids.
- 2. (144.71 sq. cm or 144.64 sq. cm.) The smaller eye should be about 14 cm. The larger eye should be about 19.5 cm. in diameter. These measurements may vary slightly. The 144.64 solution was determined using 3.14 for pi. The 144.71 solution was determined using the pi key on the calculator.
- 3. (174 3/8 or 174.375 gallons) 90 x 8 oz. = 720 oz. per day x 31 days in May = 22,320 total oz. produced divided by 128 oz. per gallon = 174.375.
- 4. (about 12 degrees. See the drawing.)



- 5. (45 m.) x/54 cm. = 25 cm./30cm. ; x/54 cm. = 5 cm./6 cm. ; x = 45 cm.
- 6. (every 30 days) The least common multiple of 2, 3, and 5 is 30.

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7. (45 minutes) This problem is best solved by drawing a diagram as follows:

At 4 mph, it would take the leader 15 minutes to finish one mile; 30 minutes for the two miles, but the rest of the parade must still go the last mile, or another 15 minutes.

1 mile	2 miles



8. (Benjie was more likely to win. 27 of 36 possible combinations would yield even number products.) To help students practice the solution strategy, have them show the possible outcomes for just three numbers on each.

					Die #1	x Die #	2 = Result	
Х	1	2	3			1	1	5 of 9 outcomes are
1	1	2	3		1	2	2	5 of 9 outcomes are
2	2	4	6	or	-	3	3	even.
3	3	6	9			1	2	
					2	2	4	
						3	6	
						1	3	
					3	2	6	
						3	9	



 $\star \star \star$ 1. Tom, John, and Bill enjoy sports. One plays tennis, one is a runner, and one is a swimmer. Tom and the tennis player are cousins. The person who runs is older than John. Bill uses the pool in his back yard daily. Which person plays each sport?



 $\star \star \star$ 2. How much does a fish weigh if its tail weighs 5 kg, its head weighs half as much as the tail and body together, and the body weighs as much as the head and tail altogether?



★ 3. Shameka said, "I think it has a mass of 25 g." Is Shameka talking about a book, a pack of chewing gum, or a paper clip?"



 \star 4. For a set of data the mean, median, and mode are as follows:

mean - 8 mode - 8 median - 8

Which set of data matches the statistics above?

8, 10, 3, 8, 7, 6, 5, 3, 3 or 7, 8, 12, 9, 8, 6, 9, 8, 5 or 6, 5, 3, 3, 7, 7, 8, 8, 8

Strategy of the Month

You have tried many ways to solve problems this year. Already you know that when one strategy does not lead you to a solution, you back up and try something else. Sometimes you can find a smaller problem inside the larger one that must be solved first. Sometimes you need to think about the information that is missing rather than what is there. Sometimes you need to read the problem again and look for a different point of view. Sometimes you need to tell your brain to try to think about the problem in an entirely different way - perhaps a way you have never used before. Looking for different ways to solve problems is like brainstorming. Try to solve this problem. You may need to change your point of view by asking, "Do all of the lines have to stay within the square formed by the dots?"

Draw 4 line segments through all 9 dots without lifting your pencil or pen.

Identifying the mathematics that is all around you can be lots of fun. Think about the geometry and spatial visualization you use in playing video games or when you play golf or basketball. When your parents parallel park, they are using their spatial skills too. When you track a hurricane, you use coordinates. When you check the stock market or read the latest sports statistics, you are using mathematics. With your family or friends go on a math scavenger hunt. Who can identify mathematics in the most unusual places?

 $\star \star \star \star$ 5. What are the possible sizes of cake squares that can be cut from a 12" x 12" pan, if no piece is smaller than 1" x 1" inches and slices are made only at whole inch measures? How many cake squares will each size provide? (Squares must be the same size.)

★★ 6. Add, subtract, multiply, and/or divide the numbers shown to get an answer of 3. (You may change the order, but you must use each number once.) Show how you got your answer in a number sentence.

7 2 13 6 3

★ 7a. How many different two-letter sequences can be made with the letters INTO? List them.

★ 7b. What is the probability of making a real English word from those two-letter sequences?

★ 7c. Would the probability of making a real English word from the letters in LOVE be greater, less, or the same? Why?

★★★ 8. The letters in MATHEMATICS are written on separate blue cards and the letters in SUPERSTAR are written on separate yellow cards. The cards are placed in a hat. What is the probability of drawing:

- 1. a vowel? _____
- 2. an S or a T? _____
- 3. an A from MATHEMATICS?_____
- 4. How much greater is the probability of drawing a blue T than a yellow T?

Setting Personal Goals

Students who recognize the value of mathematics are well on their way to becoming mathematically powerful citizens. Valuing mathematics means that we appreciate the richness, power, and usefulness of mathematics. Without math there would be no roads or bridges, computers or movies, banks or fast food restaurants. How can you become mathematically powerful?



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Discussion of problems.....

1. (Tom plays tennis, John runs, and Bill swims) Students can find the answers by making a table or matrix to keep track of clues:

	Swim	Tennis	Run
Tom		X	
John			Х
Bill			

2. (30 kg) Students may use models such as cubes or counters to represent the parts of the fish's body. By guessing and testing, they can easily conclude that if represents 5 kg (the tail), then represents 10kg (the head) and represents 15 kg (the body), and altogether the fish weighs represents 10 kg. This problem may also be solved using variables:

Let B = weight of body H = weight of head T = 5 (weight of tail)

$$H = \frac{[H+5]+5}{2}$$
 (substitution for B)

2H = H + 10 (multiply both sides by 2)H = 10 (subtract H from both sides)B = 10 + 5 = 15B + H + T = 30

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- 3. (**pack of chewing gum**) Considering the choice that are given students should realize that a pack of chewing gum is the best response.
- 4. (7, 8, 12, 9, 8, 6, 9, 8, 5) Students will examine each data set to determine the mean (average), mode (most frequent number in the data set), and median (middle number of the set of data when data are ordered from least to greatest).

5.	(See the chart)	Size	Number of cake squares
		12 x 12	1
		6 x 6	4
		4 x 4	9
		3 x 3	16
		2 x 2	36
		1 x 1	144

6. (Answers may vary--some possible answers are included)

7 x 2 - 13 + 6/3 = 3(2 -13/(7 + 6)) x 3 = 3 3 x 6/2 - (13 - 7) = 3

- (A: in, it, io, nt, no, to, ni, ti, oi, tn, on, ot = 12 combinations. B: 5/12. C: Less one word (vole) can be formed from those letters.) Students may be concerned as to whether the order of the letters matters. In spelling, it does, so "it" is counted separately from "ti."
- 8. (A: 7/20; B: 6/20 or 3/10; C: 2/20 or 1/10; D: twice as great. There is a 2:1 ratio between ` them.) Encourage students to experiment by following the directions.